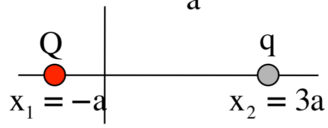
Problem 23.24

$$\left|\vec{E}(x=0)\right| = 2k\frac{Q}{a^2}$$



We know the electric field magnitude but not direction at the origin. Clearly we have two options, one in which the direction is to the right and one in which the direction is to the left.

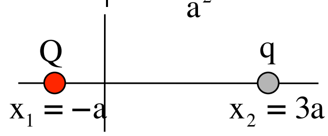
We know for sure the magnitude and direction of the charge at "-a." That is:

$$\vec{E} = k\frac{Q}{a^2}\hat{i}$$

To secure a vector in the +x direction with the appropriate magnitude, "q" would have to attract a positive test charge placed at the origin (that is, it would have to produce an electric field in the +x direction at the origin) which means it would have to be a negative charge. Additionally, the sum of its magnitude along with the magnitude of the charge Q would have to equal

$$\left|\vec{E}\right| = 2k\frac{Q}{a^2}$$

$$\left|\vec{E}(x=0)\right| = 2k\frac{Q}{a^2}$$



Messing with the math yields:

$$E_Q + E_q = 2k\frac{Q}{a^2}$$

$$k\frac{Q}{a^2} + k\frac{q}{(3a)^2} = 2k\frac{Q}{a^2}$$

$$\Rightarrow \quad q = 9Q$$

(Again, this value is predicated on the assumption that "q" is negative.)

If we assume "q" is positive, then the two fields will subtract from one another ("q," being positive, will produce a field to the left, fighting the right-directed field produced by "Q"). In that case:

$$E_Q - E_q = 2k\frac{Q}{a^2}$$

$$k\frac{Q}{a^2} - k\frac{q}{(3a)^2} = 2k\frac{Q}{a^2}$$

$$\Rightarrow \quad q = 27Q$$